Probing T-Branes

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hep-th/1006.5459 w/ C. Vafa

hep-th/1009.0017 w/ Y. Tachikawa, C. Vafa and B. Wecht

hep-th/1010.5780 w/ S. Cecotti, C. Cordova and C. Vafa

Plan of the Talk

- Motivation

- $\cap$ 7-Branes and T-Branes

- Probing T-Branes with D3-Branes

- Conclusions

Motivation

A common theme in string theory is:

- Geometry leads to novel field theories
- Probe theories useful to study geometry

This set of questions is of intrinsic interest

Also relevant for stringy model building

F-theory

An example of this type is F-theory

Access to string theory with $\tau_{IIB} \sim O(1)$

Specify (in part) via Weierstrass model:

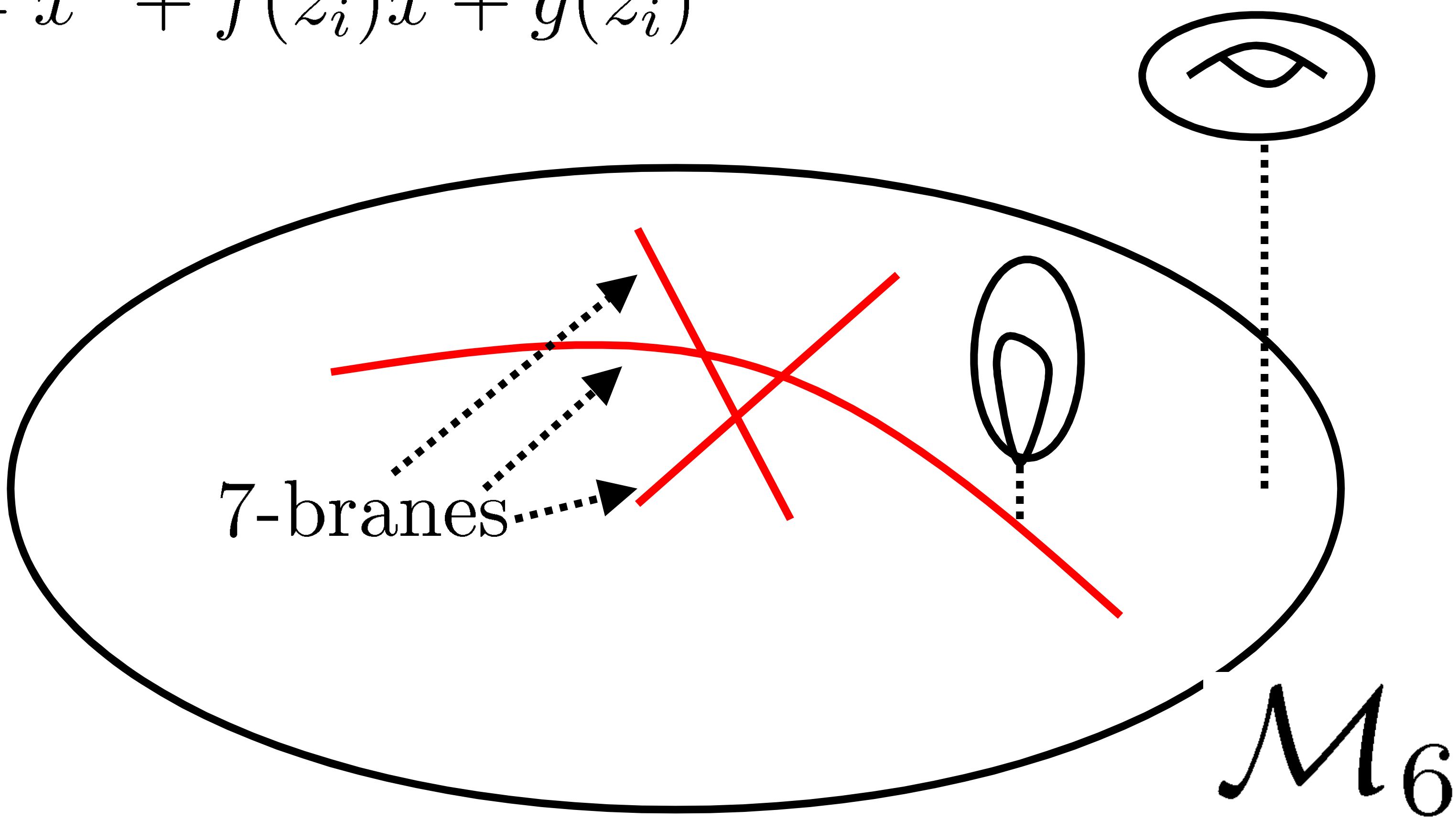
$$y^2 = x^3 + f(z_i)x + g(z_i)$$

7-Branes where $\Delta = 4f^3 + 27g^2 = 0$

Geometric Formulation

F / $CY_4 \Rightarrow$ 4D $\mathcal{N} = 1$ SUSY

$$y^2 = x^3 + f(z_i)x + g(z_i)$$



7-Brane Gauge Theory

7-brane wrapping $\mathbb{R}^{7,1} \Rightarrow$ 8D SYM

Gauge gp dictated by F-theory Geometry

Examples:

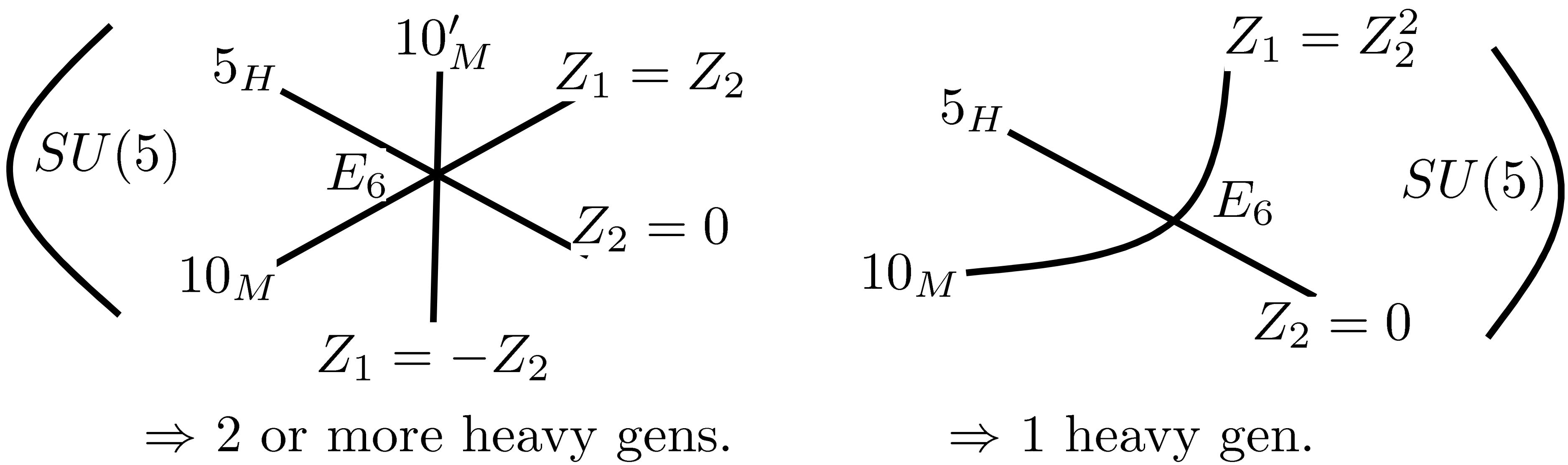
$$y^2 = x^3 + z^5 \Rightarrow E_8 \text{ gauge gp.}$$

$$y^2 = x^3 + z^4 \Rightarrow E_6 \text{ gauge gp.}$$

7-brane at $(z = 0)$

$\cap$ 7-Branes and Fenomenology

In model building applications, the details of $\cap$'s strongly determine the phenomenology



$\Rightarrow$ Need to know class of possible $\cap$'s

The Main Idea

However, 8D SYM has more to it
than just the F-theory geometry

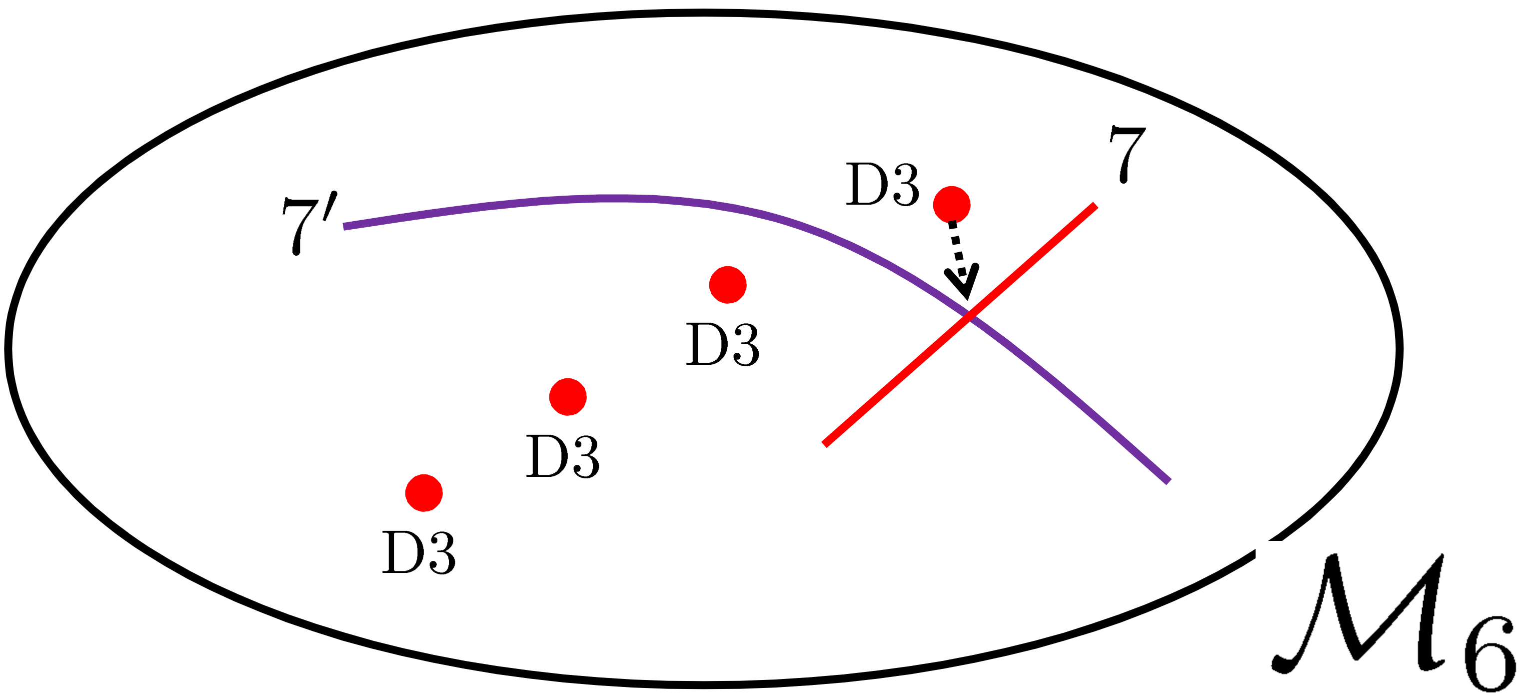
Physical ambiguities in Weierstrass model

In this talk study using:

- 8D SYM
- probe D3-branes

Probing ∩ 7-Branes I

Question: What does a D3-brane see?



Probing $\cap$ 7-Branes II

Details of 7-Brane $\cap$'s determine D3 probe theory

D3 probes of such $\cap$'s can lead to:

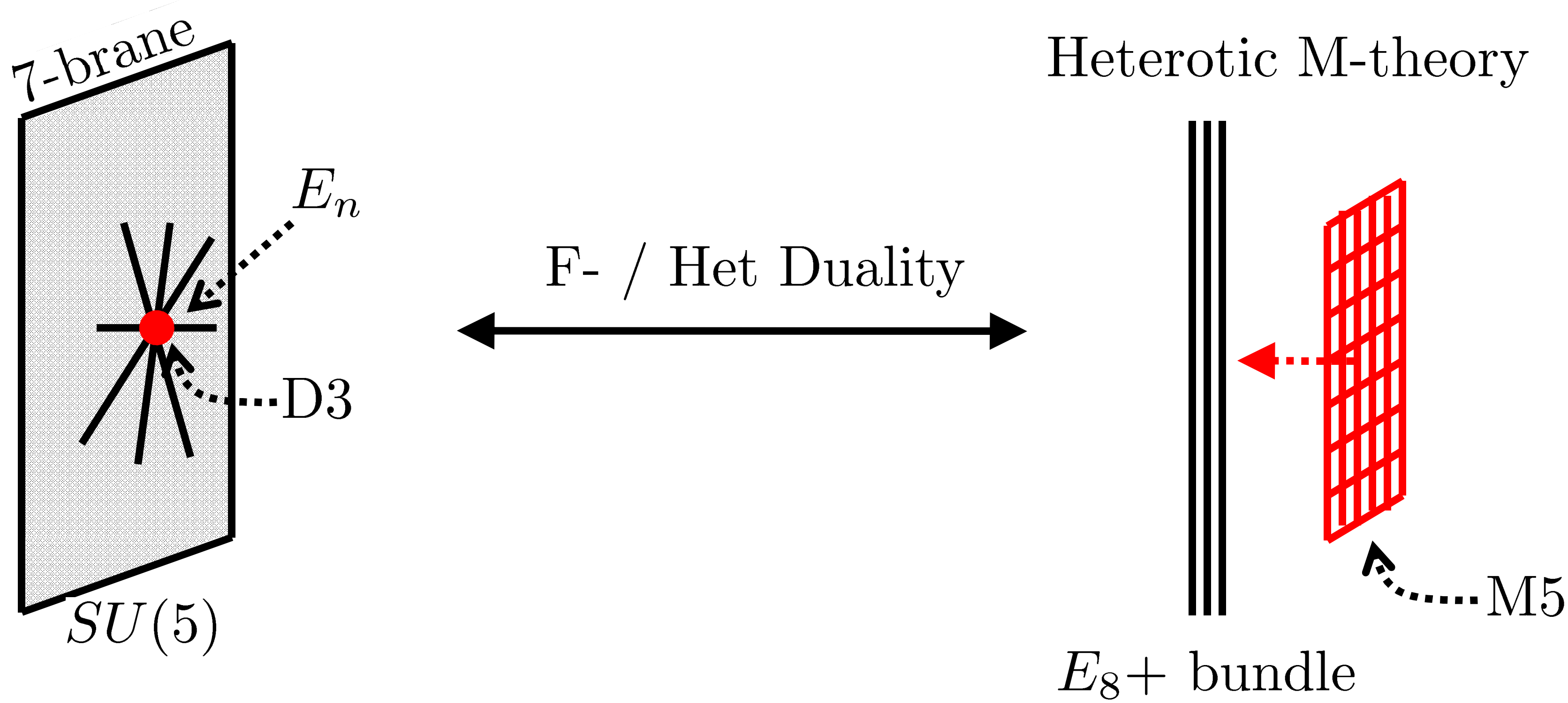
- Additional info. on candidate 7-Brane $\cap$'s
- New non-Lagrangian $\mathcal{N} = 1$ SCFTs
- Source of novel extra sectors

Model Building?

E-type ∩'s also of relevance for GUT models

Beasley JJH Vafa '08, Donagi Wijnholt '08, Hayashi et al. '08

D3-Branes = novel extra sector



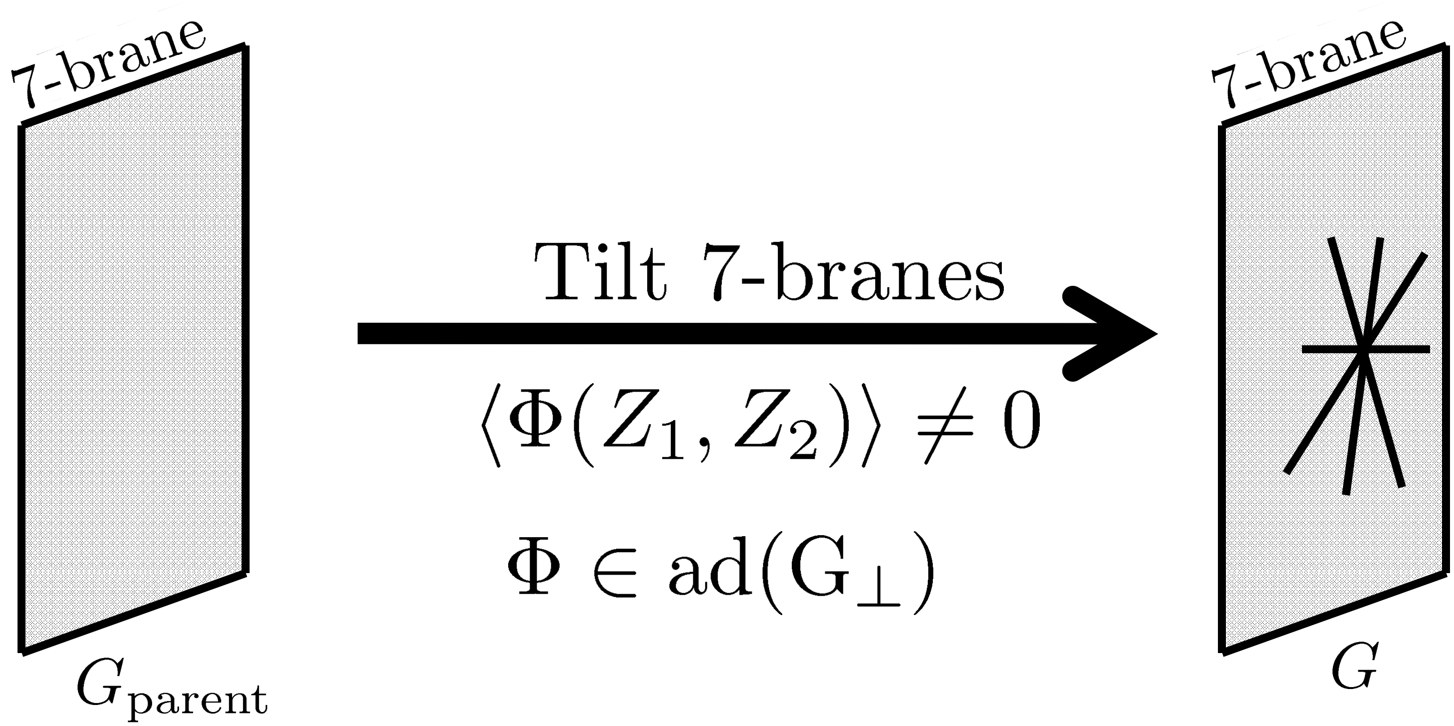
Roadmap

- Motivation

- $\cap$ 7-Branes and T-Branes

Basic Picture

Tilting via breaking pattern $G_{\text{parent}} \rightarrow G \times G_{\perp}$:



$\cap$ 7-Branes

Katz Vafa '96
Beasley JJH Vafa '08
Donagi Wijnholt '08

Local $\cap$'s of 7-Branes from 8D SYM on $\mathbb{R}^{3,1} \times S$

Field content $\supset$

$$A_{(0,1)} = \text{gauge field}$$

$$\Phi_{(2,0)} \text{ in adjoint of } G_{\text{parent}}$$

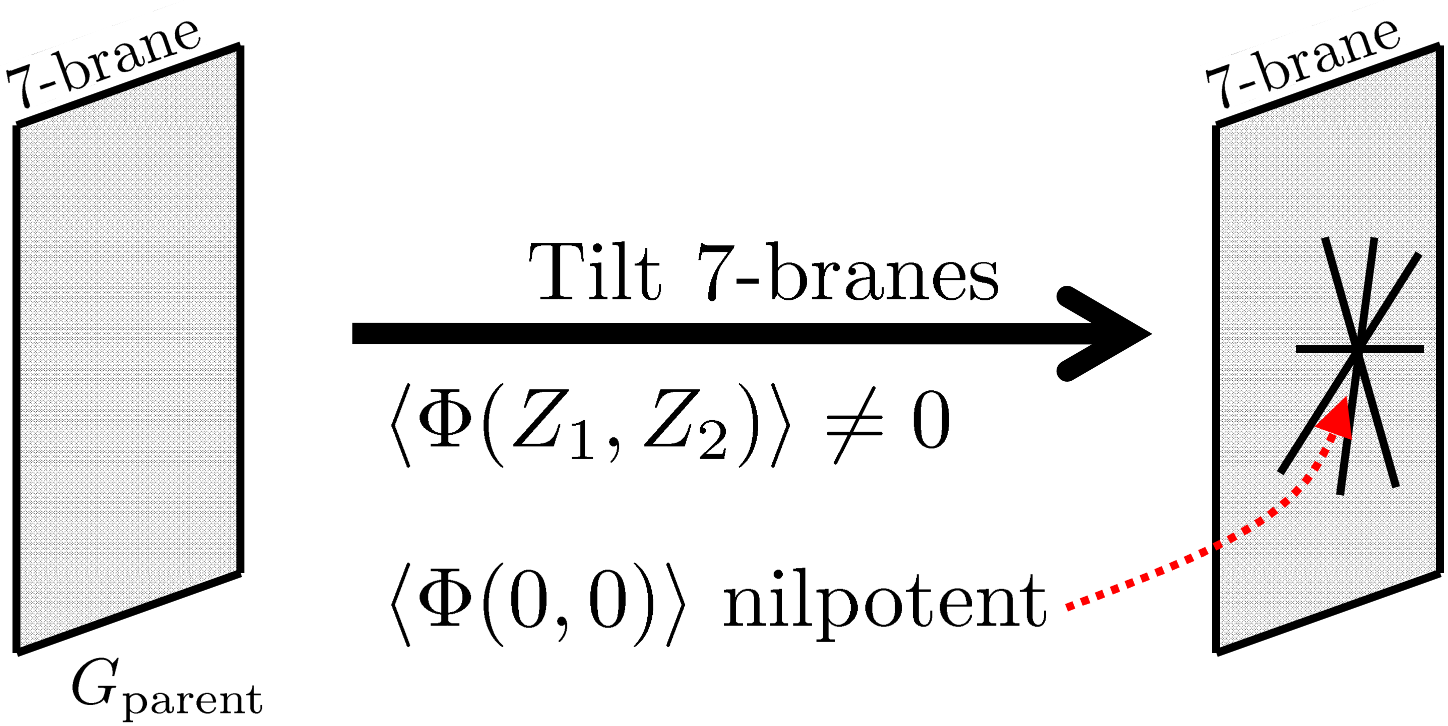
EOMs on S:

$$F^{(0,2)} = F^{(2,0)} = \overline{\partial}_A \Phi = 0$$

$$\omega \wedge F_{(1,1)} - \frac{i}{2}\left[\Phi, \Phi^{\dagger}\right] = 0$$

Tilting 7-Branes

Position of 7-Branes dictated by $\Phi \in \mathrm{ad}(\mathrm{G}_{\mathbb{C}})$



Breaking Patterns

In this talk focus on $G_{\perp} = SU(n)$

Tilting specified by $\Phi(Z_1, Z_2)$ an $n \times n$ matrix

Heuristically: $\mathrm{Eigen}(\Phi) =$ 7-Brane Positions

Note: $\mathrm{Eigen}(\Phi)$ has Z_i dependence

Monodromy

Generically, Eigen(Φ) has branch cuts

e.g. "monodromy"

$$\Phi^n + b_1(Z_1, Z_2)\Phi^{n-1} + \cdots + b_n(Z_1, Z_2) = 0$$

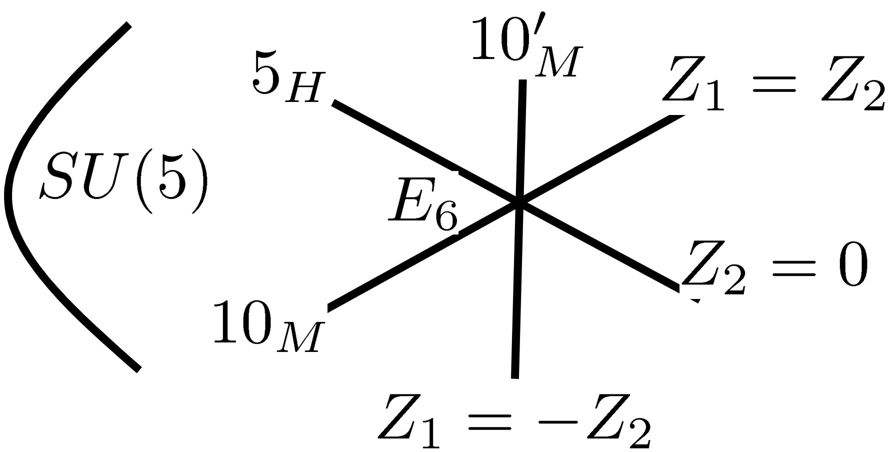
Example: $\Phi^2 - Z_1 = 0$

Model Building Example

Hayashi et al. ’09; Cecotti Cordova JJH Vafa ’10

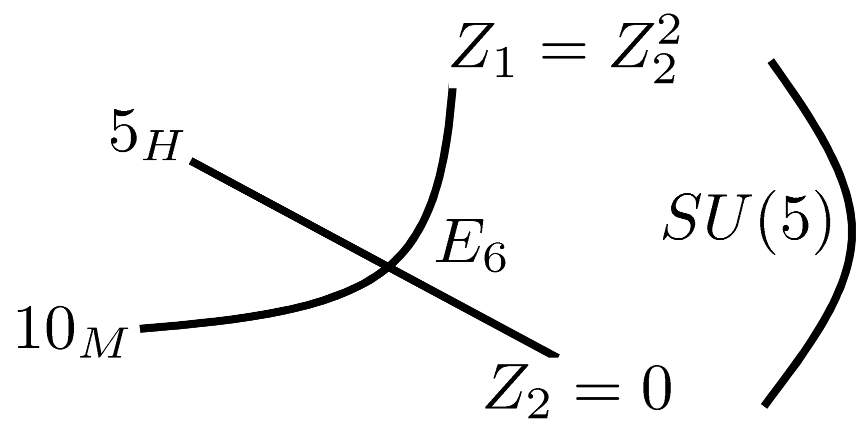
Unfolding $E_6 \rightarrow SU(5) \times SU(2) \times U(1)$

$$\Phi = \# \begin{bmatrix} Z_1 & \\ & -Z_1 \end{bmatrix} \oplus Z_2$$



$\Rightarrow$ 2 or more heavy gens.

$$\Phi = \# \begin{bmatrix} & 1 \\ Z_1 & \end{bmatrix} \oplus Z_2$$



$\Rightarrow$ 1 heavy gen.

Unfolding E_8

$$\text{Eigenvalues}(\Phi(Z_1, Z_2)) = \text{7-Brane ``Positions''}$$

$$E_8 \to SU(5)_{GUT} \times SU(5)_\perp \qquad \Phi \in \mathrm{ad}(\mathrm{SU}(5)_\perp)$$

$$b_0\Phi^5 + b_2(Z_1, Z_2)\Phi^3 + \cdots + b_5(Z_1, Z_2) = 0$$

Hayashi et al. ’09
Donagi Wijnholt ’09

$$y^2 = x^3 + b_0 z^5 + b_2 x z^3 + b_3 y z^2 + b_4 x^2 z + b_5 x y$$

valid in a local patch

Specifying The Holomorphic Data

$$y^2 = x^3 + b_0 z^5 + b_2 x z^3 + b_3 y z^2 + b_4 x^2 z + b_5 x y$$

Just specifying b_i does *not* fix physical theory

Φ is a matrix, b_i just five invariants

Ambiguity?

$$y^2 = x^3 + b_0 z^5 + b_2 x z^3 + b_3 y z^2 + b_4 x^2 z + b_5 x y$$

$$y^2 = x^3 + b_0 z^5$$

¿¿¿$\Phi = 0$ versus Φ nilpotent???

T-Branes

Cecotti, Cordova, JJH, Vafa '10
see also Katz Donagi Sharpe '03

T-Branes: Φ which is nilpotent at some (Z_1, Z_2)

e.g. looks "upper triangular"

Constant part of Φ can be put in Jordan form:

$$\Phi^{(0)} = \begin{bmatrix} \lambda_1^{(1)} & 1 & 0 & 0 \\ 0 & ... & ... & 0 \\ 0 & 0 & ... & 1 \\ 0 & 0 & 0 & \lambda_{N_1}^{(1)} \end{bmatrix} \oplus ... \oplus \begin{bmatrix} \lambda_1^{(a)} & 1 & 0 & 0 \\ 0 & ... & ... & 0 \\ 0 & 0 & ... & 1 \\ 0 & 0 & 0 & \lambda_{N_a}^{(a)} \end{bmatrix}$$

In a T-brane, $\lambda_i^{(k)} = 0$

Roadmap

- $\cap$ 7-Branes and T-Branes

- Probing T-Branes with D3-Branes

D3-Brane Probe

D3 probe of $\cap$ E-type 7-Branes leads to

Strongly coupled $\mathcal{N} = 1$ theory

Can view as $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ deformation

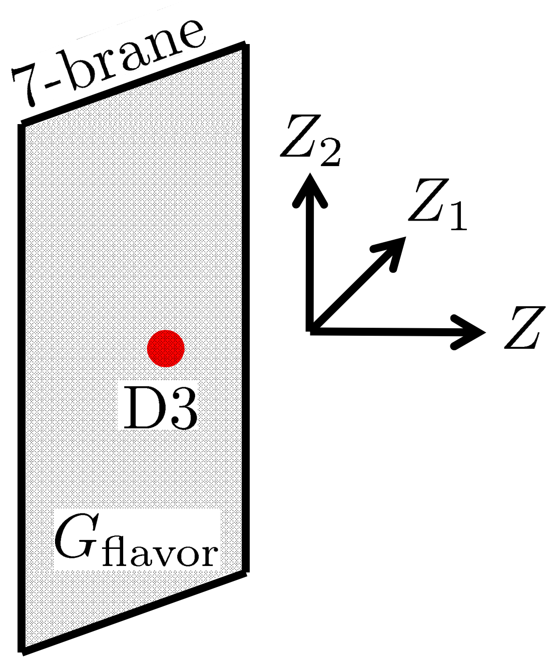
Plan for remainder of talk:

- Review $\mathcal{N} = 2$ case
- Study $\mathcal{N} = 1$ deformations

Warmup: $\mathcal{N} = 2$ Probes

D3-brane probing parallel stack of 7-branes

Banks Douglas Seiberg '96,
Douglas Lowe Schwarz '96, …

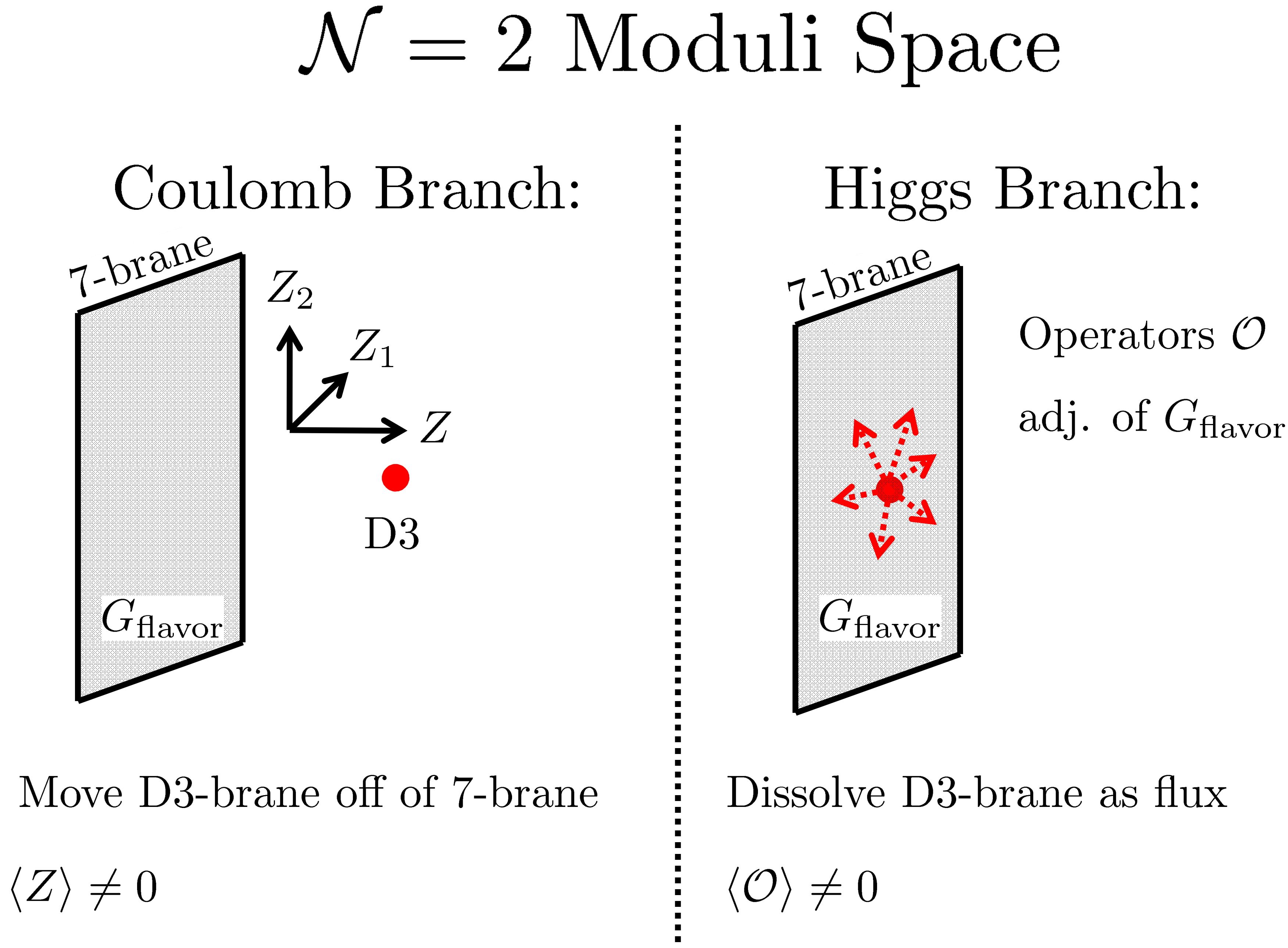


7-brane gauge group $= G_{\text{flavor}}$

$3-3$ strings: Z_i and Z

$3-7$ string composite operators $\mathcal{O}_{\text{adj}}$

(analogue at weak coupling: $\mathcal{O} \sim Q\widetilde{Q}$)


$\mathcal{N} = 2$ Moduli Space
Coulomb Branch:
7-brane
Z_2
Z_1
Z
D3
G_{flavor}
Move D3-brane off of 7-brane
$\langle Z \rangle \neq 0$
Higgs Branch:
7-brane
Operators $\mathcal{O}$
adj. of G_{flavor}
G_{flavor}
Dissolve D3-brane as flux
$\langle \mathcal{O} \rangle \neq 0$

$\mathcal{N} = 2$ SCFTs

Specific F-th singularities $\Rightarrow$ constant τ

Sen '96, Banks Douglas Seiberg '96, Dasgupta Mukhi '96,…

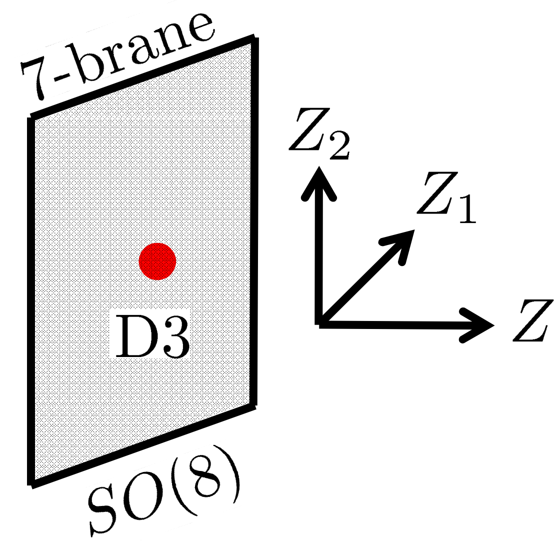
D3-probe = strongly coupled $\mathcal{N} = 2$ SCFT

	H_0	H_1	H_2	D_4	E_6	E_7	E_8
$\Delta(Z)$	$\frac{6}{5}$	$\frac{4}{3}$	$\frac{3}{2}$	2	3	4	6

"Argyres-Douglas" "Minahan-Nemeschansky"

Example: $SO(8)$ probe

D3 probe of $SO(8)$ 7-Brane:



Field Content: $SU(2)$ SYM $\oplus \varphi \oplus 4 \times (Q_i \oplus \widetilde{Q}_i)$

$W_{\mathcal{N}=2} = \sum_{i=1}^{i=4} Q_i \cdot \varphi \cdot \widetilde{Q}_i$

Coul branch: $z = \langle \mathrm{Tr}\varphi^2 \rangle$ Higgs branch: $\langle QQ' \rangle$

$\mathcal{N} = 2$ Curve for $SO(8)$ probe

$$y^2 = x^3 + xz^2 + Az^3$$

Sen '96, Banks Douglas Seiberg '96

Seiberg-Witten Curve = F-theory Geometry!

Very useful, curve encodes:
- $\tau_{U(1)} = \tau_{IIB}$
- BPS masses
- Scaling dim^{n}s

$\mathcal{N}=2$ E_n Probes

Minahan-Nemeschansky: Introduce $\mathcal{N}=2$ SCFT

Minahan Nemeschansky '96

Seiberg-Witten Curves:

$$E_8 : y^2 = x^3 + z^5$$

$$E_7 : y^2 = x^3 + xz^3$$

$$E_6 : y^2 = x^3 + z^4$$

$$\mathcal{N}=2 \text{ D3-probe} = \text{MN}_{\mathcal{N}=2} \oplus (Z_1 \oplus Z_2)$$

Some properties:

- $\tau \sim O(1)$ on Coulomb Branch

- $\Delta(Z)$

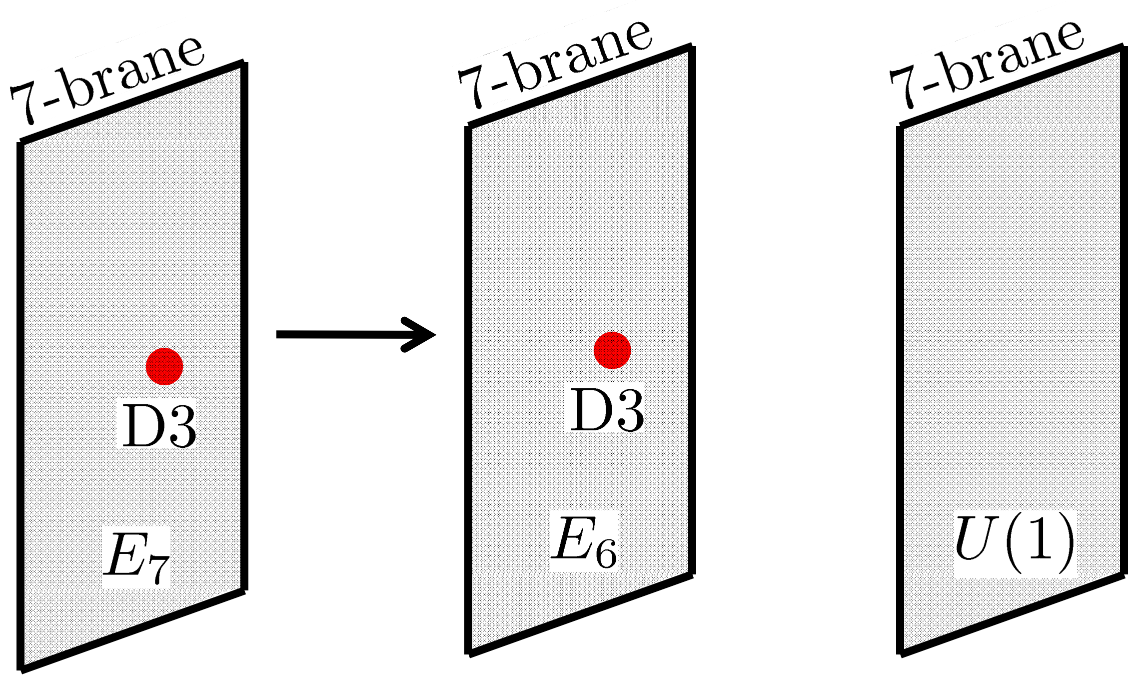
	E_6	E_7	E_8
$\Delta(Z)$	3	4	6

$\mathcal{N} = 2$ Deformations

Deformations: $\delta\mathcal{L} = \int d^2\theta \ \mathrm{Tr}_{E_n}(\Phi \cdot \mathcal{O}_{\mathrm{adj}}) + \mathrm{h.c.}$

Φ constant and $[\Phi, \Phi^\dagger] = 0$

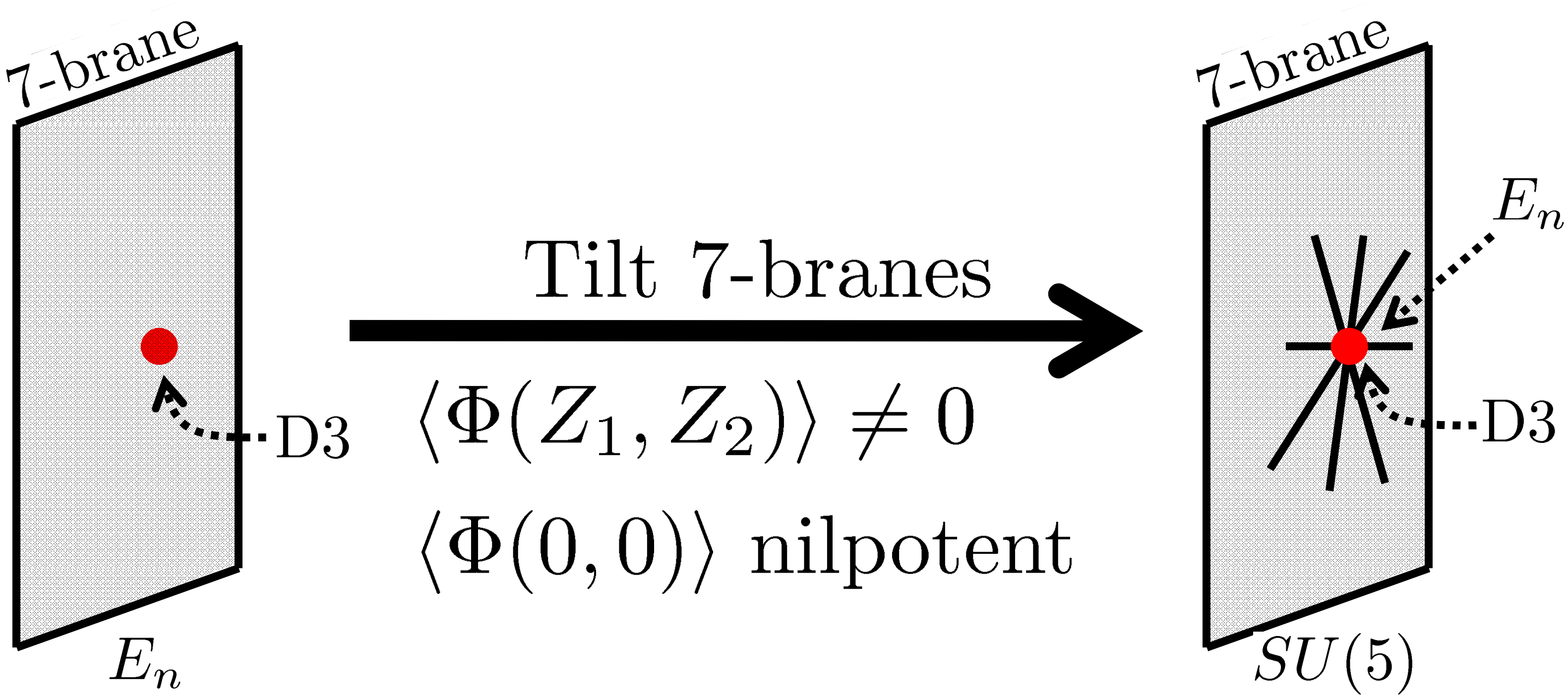
Moves 7-branes:



Probing an E-point

$\mathcal{N} = 2$ SCFT

$\mathcal{N} = 1$ Deform^n



$$\mathcal{N} = 2 \to \mathcal{N} = 1$$

$$\delta\mathcal{L} = \int d^2\theta \ \mathrm{Tr}_{E_n}(\Phi(Z_1, Z_2) \cdot \mathcal{O}_{\mathrm{adj}}) + \mathrm{h.c.}$$

JJH Vafa ’10 (see also Aharony Kachru Silverstein ’96)

$$\text{T-Brane} \Rightarrow [\Phi, \Phi^\dagger] \neq 0$$

In T-brane $\Phi^{(0)}$ = sum of nilp. Jordan Blocks

$\Phi^{(0)} = 0$: Flows back to $\mathcal{N} = 2$ theory

Follows from Green et al. ’10

$\Phi^{(0)} \neq 0$: Can flow to new $\mathcal{N} = 1$ SCFTs

JJH Tachikawa Vafa Wecht ’10

Nilpotent Deformations

Recall: T-Brane has $\Phi(Z_i = 0)$ nilpotent

Even Φ constant and nilpotent is interesting

$$y^2 = x^3 + b_0 z^5 + b_2 x z^3 + b_3 y z^2 + b_4 x^2 z + b_5 x y$$

$b_i =$ Casimirs of $\Phi \Rightarrow$ F-th geometry stays put:

$$y^2 = x^3 + b_0 z^5$$

But: Probe theory changes! $\delta W = \mathrm{Tr}_{E_n}(\Phi \cdot \mathcal{O}_{\mathrm{adj}}) \neq 0$

IR R-Symmetry

If a CFT, we can determine some details of IR

By computing IR R-symmetry:

$$R_{IR} = R_{UV} + \sum t_I R_I$$

← $U(1)$ flavor symmetries

Maximize over $a(t_I) = \frac{3}{32}[3TrR_{IR}^3 - TrR_{IR}]$

Intriligator Wecht '03

Note: Just need *anomalies*

Example: $SO(8)$ probe

JJH Tachikawa Vafa Wecht '10

$$Tr\,(\Phi\cdot O) = \begin{bmatrix} Q_1 & Q_2 \end{bmatrix} \begin{bmatrix} 0 & m_{1\overline{2}} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \widetilde{Q}_{\overline{1}} \\ \widetilde{Q}_{\overline{2}} \end{bmatrix}$$

$$W = \sum_{1\leq i\leq 4} Q_i\cdot\varphi\cdot\widetilde{Q}_i + m_{1\overline{2}} Q_1 \widetilde{Q}_{\overline{2}}$$

$$W_{eff} = \sum_{3\leq i\leq 4} Q_i\cdot\varphi\cdot\widetilde{Q}_i - \frac{Q_2\varphi^2\widetilde{Q}_{\overline{1}}}{m_{1\overline{2}}}$$

$\Delta_{IR}(Z)\sim 1.5$ (Determine via a-maximization)

Non-Lagrangian Case

$\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ via $\delta W = Tr_{E_n}(\Phi \cdot O)$

Even though non-Lagrangian in UV and IR

- We know the flavor symmetries

  (assume no accidental $U(1)$'s)

- We know anomalies in $\mathcal{N} = 2$ UV theory

$\Rightarrow$ We can still fix R_{IR}

JJH, Tachikawa, Vafa, Wecht '10

UV Symmetries

$U(1)_R^{\mathcal{N}=2} \times SU(2)_R$:

- $R_{UV}^{\mathcal{N}=1} = \frac{1}{3} R_{UV}^{\mathcal{N}=2} + \frac{4}{3} I_3$
- $J_{\mathcal{N}=2} = R_{UV}^{\mathcal{N}=2} - 2I_3$

Rotations of Z_i:

- J_i (neglect in nilp. case)

Cartan of E_n:

- $J = \oplus_j \mathrm{diag}(\mathrm{j}, ..., -\mathrm{j})$

$j = \frac{n-1}{2}$ for $n \times n$ nilp. block of Φ

IR R-Symmetry

$$R(t) = R_{UV} + (\tfrac{t}{2} - \tfrac{1}{3})J_{\mathcal{N}=2} - tJ + \mu_1(t)J_1 + \mu_2(t)J_2$$

set by details of $\Phi(Z_1, Z_2)$

$$a(t) = \tfrac{3}{32}(3R(t)^3 - R(t))$$

- $R(t)^3 =$ sum of UV cubic anomalies
- $R(t) =$ sum of UV linear anomalies

Anomaly Matching

Even though non-Lagrangian in UV

We know anomalies of $\mathcal{N}=2$ theory

Ganor et al. '97, Argyres Seiberg '07, Aharony Tachikawa '07, Argyres Wittig '08

$$a_{UV} = \frac{3}{32}[3TrR_{UV}^3 - TrR_{UV}] = \frac{3}{4}\Delta_{UV}(Z) - \frac{1}{2}$$

$$c_{UV} = \frac{1}{32}[9TrR_{UV}^3 - 5TrR_{UV}] = \Delta_{UV}(Z) - \frac{3}{4}$$

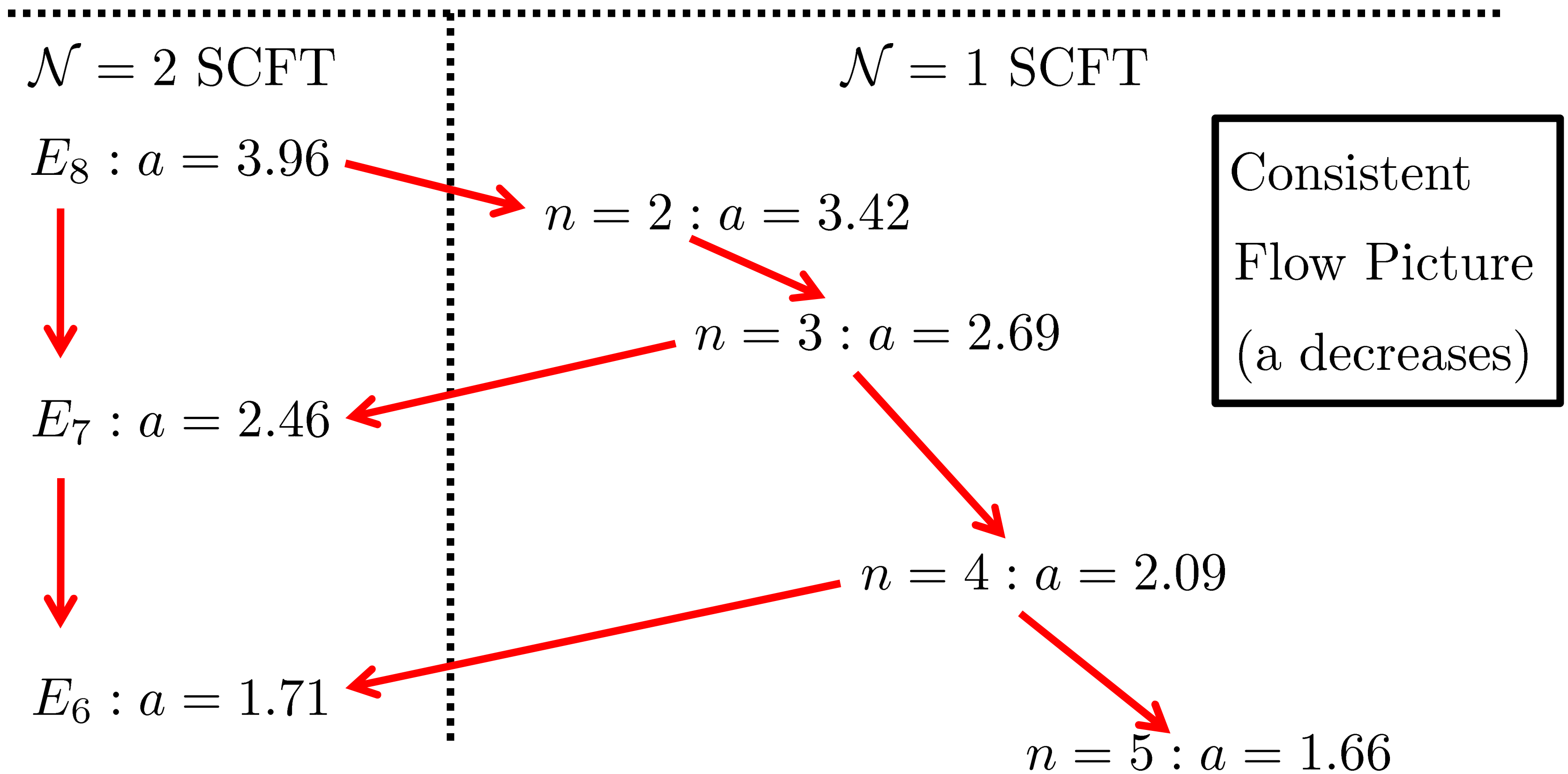
$$k_{UV} = -\frac{1}{6}[TrR_{UV}J^{flav}J^{flav}] = 2\Delta_{UV}(Z)$$

$\Rightarrow$ Enough to determine R_{IR}

JJH, Tachikawa, Vafa, Wecht '10

Nilpotent Families

$$\Phi^{(n)} = \begin{bmatrix} 0 & m_{1\bar{2}} & 0 & 0 \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & m_{n\overline{n-1}} \\ 0 & 0 & 0 & 0 \end{bmatrix} \in SU(n) \subset SU(9) \subset E_8$$



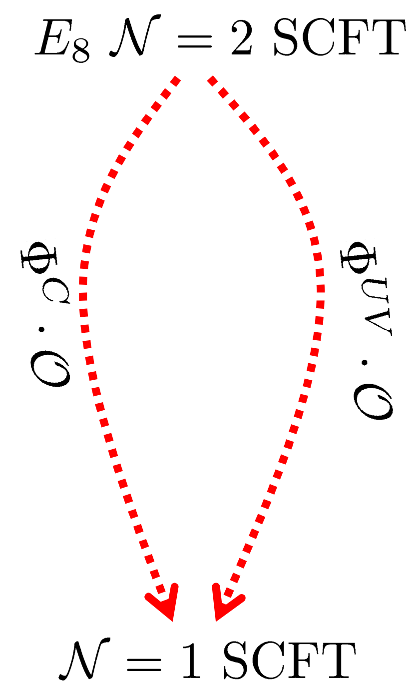
Coarse-Grained T-Branes

Making Φ depend on $Z_i \Rightarrow$ field dep. mass terms

Most Details wash out in IR

$$\Phi^{UV} = \begin{bmatrix} f_{1\bar{1}}(Z_i) & m_{1\bar{2}} & 0 & 0 \\ f_{2\bar{1}}(Z_i) & f_{2\bar{2}}(Z_i) & \ldots & 0 \\ \ldots & \ldots & \ldots & m_{n\overline{n-1}} \\ f_{n\bar{1}}(Z_i) & \ldots & \ldots & f_{n\bar{n}}(Z_i) \end{bmatrix}$$

$$\Phi^{C} = \begin{bmatrix} 0 & m_{1\bar{2}} & 0 & 0 \\ 0 & 0 & \ldots & 0 \\ \alpha Z_2 & 0 & 0 & m_{n\overline{n-1}} \\ Z_1 & \beta Z_2 & 0 & 0 \end{bmatrix}$$



same IR fixed point

Roadmap

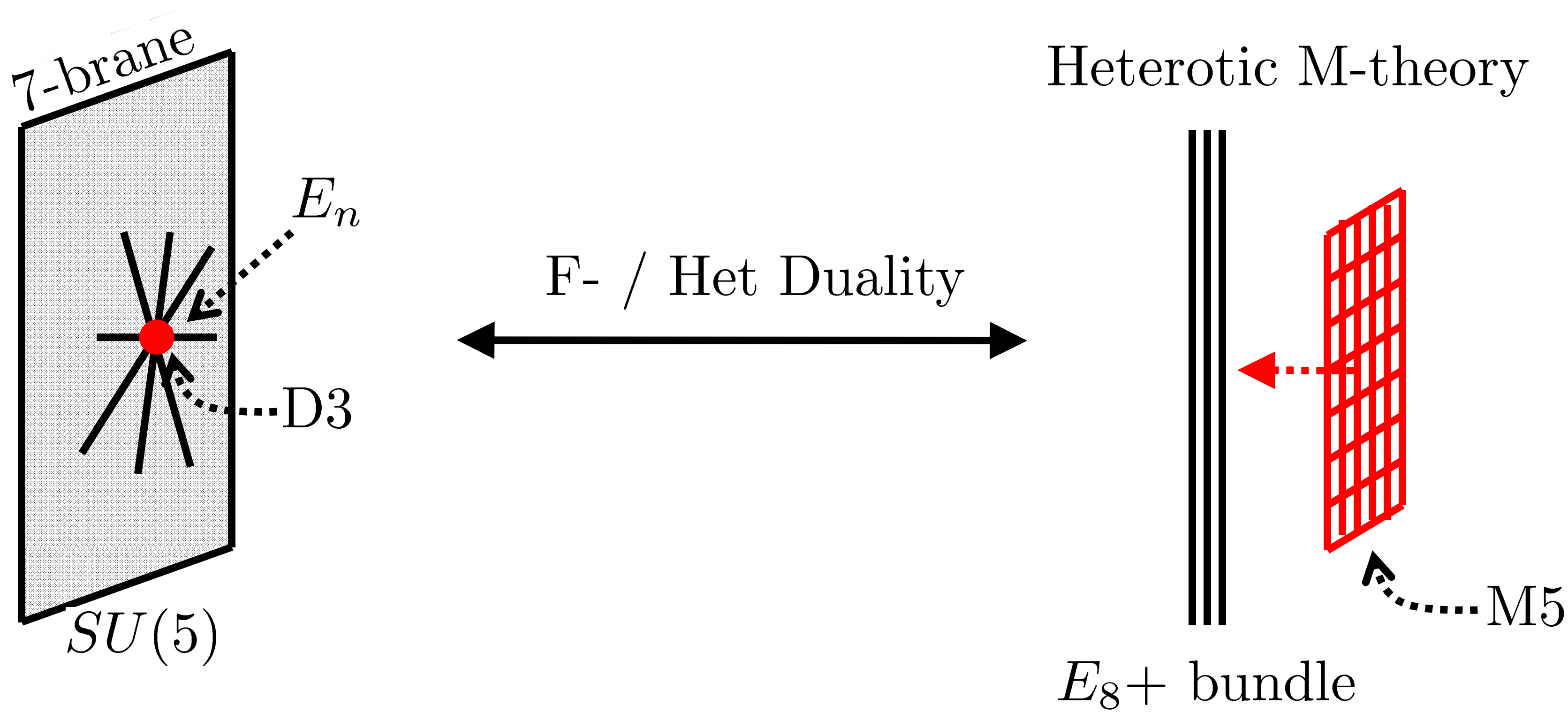
- Probing T-Branes with D3-Branes

- Future Directions / Conclusions

Model Building?

View ∩ 7-Branes as Standard Model sector

D3-Branes = novel extra sector



Visible Sector Couplings

$\exists$ CFT states charged under SM gauge group

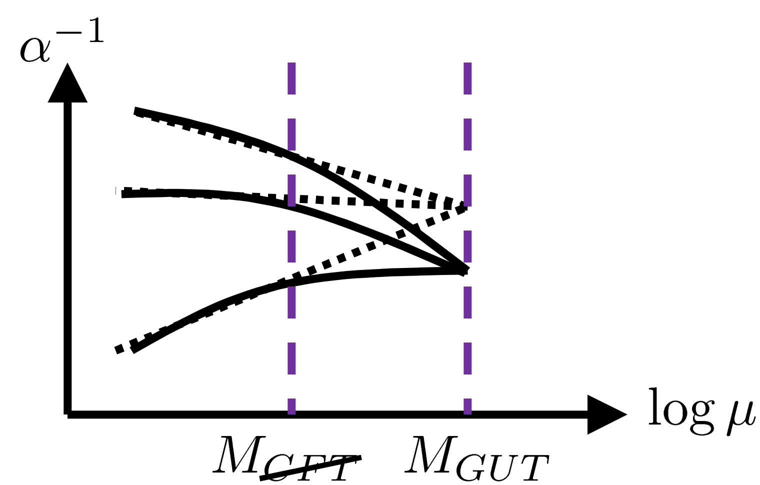
$\Rightarrow$ CFT must be broken at scale $M_{\cancel{CFT}} > M_{\text{weak}}$

Coupling to matter: $\int d^2\theta \; \Psi_R \mathcal{O}_{R^*}$

Also couples to gauge fields

irrat^l # of "particles"

$\sim$ two to six $5 \oplus \overline{5}$'s



Applications?

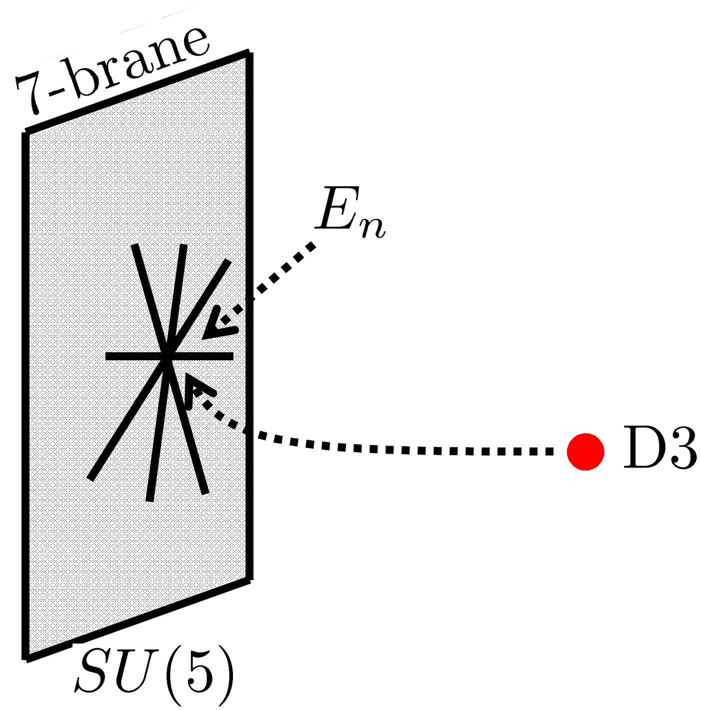
Phenomenology looks quite rich (and unexplored)

As an inflaton?

~~SUSY~~? Dark Matter?

Collider Signatures?

$+\cdots ?$



In Progress: JJH Rey; JJH Vafa Wecht

Conclusions

- T-Branes: Generalization of $\cap$ 7-Branes
- D3-probes of T-branes $\Rightarrow$ novel $\mathcal{N} = 1$ SCFTs
- Broad class of $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ deformations
- ¿Model building with D3-branes?